3.1 – Vectors in 2-Space, 3-Space, and n-Space

Definition: If n is a positive integer, then an **ordered n-tuple** is a sequence of n real numbers $(v_1, v_2, \ldots, v_n)$. The set of all ordered n-tuples is called **real n-space**, denoted by R^n.

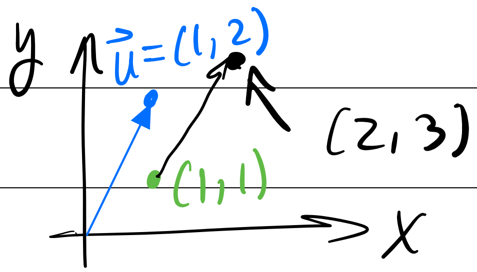
$\mathbb{R}^n$

If a vector **v** in 2-space or 3-space is positioned with its initial point at the origin of a rectangular coordinate system, then the coordinates of its terminal point are **components** of **v** relative to the coordinate system. The numbers in an n-tuple can be considered coordinates of a point or components of a vector.

#5 a. Find the terminal point of the vector that is equivalent to $\mathbf{u} = (1,\ 2)$ and whose initial point is $A(1,\ 1)$.
b. Find the initial point of the vector that is equivalent to $\mathbf{u} = (1,\ 1,\ 3)$ and whose terminal point is $B(-1,\ -1,\ 2)$.

terminal initial

a.



Copy of $\vec{u}$: $(2-1, 3-1) = (1,2)$

b. $(-1-x_1, -1-y_1, 2-z_1) = (1,1,3)$

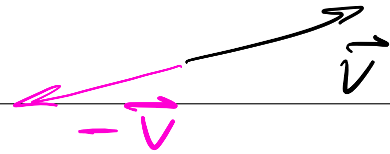
$x_1 = -2,\ x_2 = -2,\ x_3 = -1$

The initial point is $(-2,-2,-1)$.

#7 Find the initial point P of a nonzero vector $\mathbf{u} = \overrightarrow{PQ}$ with terminal point $Q(3,\ 0,\ -5)$ and such that

a. $\mathbf{u}$ has the same direction as $\mathbf{v} = (4,\ -2,\ -1)$. — Same as previous

b. $\mathbf{u}$ is oppositely directed to $\mathbf{v} = (4,\ -2,\ -1)$.



$-\vec{v} = (-4, 2, 1)$, then same as previous

Definition: If $\mathbf{v} = (v_1, v_2, \ldots,\ v_n)$ and $\mathbf{w} = (w_1, w_2, \ldots,\ w_n)$ are vectors in R^n, and if k is any scalar, then we define

- $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \ldots,\ v_n + w_n)$ [this is **vector addition**]
- $k\mathbf{v} = (kv_1, kv_2, \ldots,\ kv_n)$ [this is **scalar multiplication**]
- $-\mathbf{v} = (-v_1, -v_2, \ldots, -v_n)$
- $\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \ldots,\ w_n - v_n)$

#**12** Let $\mathbf{u} = (1,\ 2,\ -3,\ 5,\ 0)$, $\mathbf{v} = (0,\ 4,\ -1,\ 1,\ 2)$, and $\mathbf{w} = (7,\ 1,\ -4,\ -2,\ 3)$. Find the components of

a. $\mathbf{v} + \mathbf{w}$

b. $3\,(2\mathbf{u} - \mathbf{v})$

c. $(3\mathbf{u} - \mathbf{v}) - (2\mathbf{u} + 4\mathbf{w})$

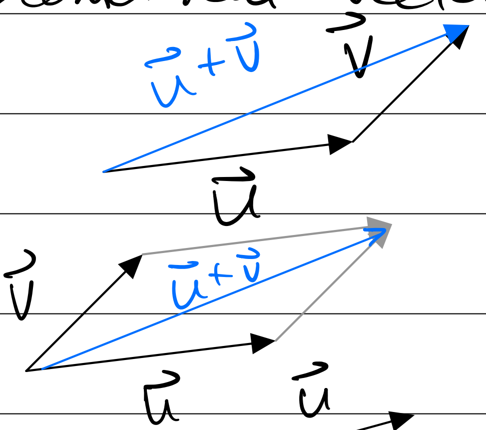
d. $\frac{1}{2}\,(\mathbf{w} - 5\mathbf{v} + 2\mathbf{u}) + \mathbf{v}$

a. $\vec{v} + \vec{w} = (0+7,\ 4+1,\ -1-4,\ 1-2,\ 2+3)$

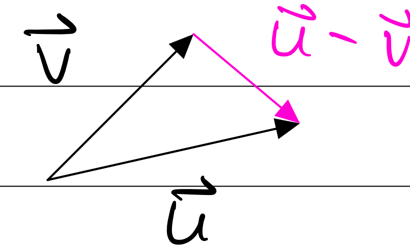
$= (7,\ 5,\ -5,\ -1,\ 5)$

practice b – d.

Geometrical vector addition



Subtraction



scalar mult:

The **zero vector**, $\mathbf{0} = (0,\ 0,\ \ldots,\ 0)$, has length zero. $\vec{0}$

Theorem 3.1.1 If **u**, **v**, and **w** are vectors in R^n, and if k and m are scalars, then:

a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) = \mathbf{u} + \mathbf{v} + \mathbf{w}$

c) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

e) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$

f) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$

g) $k(m\mathbf{u}) = (km)\mathbf{u}$

h) $1\mathbf{u} = \mathbf{u}$

When we use properties of real #s, we need to use real numbers

This proof uses components of vectors

pf (f): Let $\vec{u} = (u_1, u_2, \ldots, u_n)$, $k, m \in \mathbb{R}$.

$(k+m)\vec{u} = ((k+m)u_1,\ (k+m)u_2,\ \ldots,\ (k+m)u_n)$

by def of scalar mult.

$= (ku_1 + mu_1,\ ku_2 + mu_2,\ \ldots,\ ku_n + mu_n)$

dist. property for real #s.

$= (ku_1, ku_2, \dots, ku_n) + (mu_1, mu_2, \dots, mu_n)$ def of vector addition

$= k\vec{u} + m\vec{u}$ def of scalar mult.

Theorem 3.1.2 If $\mathbf{v}$ is a vector in R^n and k is a scalar, then:

a) $0\mathbf{v} = \mathbf{0}$

b) $k\mathbf{0} = \mathbf{0}$

c) $(-1)\mathbf{v} = -\mathbf{v}$

Definition: if $\mathbf{w}$ is a vector in R^n, then $\mathbf{w}$ is said to be a **linear combination** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r$ in R^n if it can be expressed in the form $\mathbf{w} = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \dots + k_r\mathbf{v}_r$, where each k_i is a scalar. The scalars are called the **coefficients** of the linear combination. In the case where $r = 1$, we have $\mathbf{w} = k_1\mathbf{v}_1$, in which case the linear combination is a scalar multiple of the vector.

#17 Let $\mathbf{u} = (1, -1, 3, 5)$ and $\mathbf{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\mathbf{u} + b\mathbf{v} = (1, -4, 9, 18)$. $= C$

$$a\vec{u} + b\vec{v} = \vec{C} \Rightarrow \begin{bmatrix} 1 \\ -1 \\ 3 \\ 5 \end{bmatrix} a + \begin{bmatrix} 2 \\ 1 \\ 0 \\ -3 \end{bmatrix} b = \begin{bmatrix} 1 \\ -4 \\ 9 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} a+2b \\ -a+b \\ 3a \\ 5a-3b \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 9 \\ 18 \end{bmatrix} \Rightarrow \begin{aligned} a+2b &= 1 \\ -a+b &= -4 \\ 3a &= 9 \\ 5a-3b &= 18 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ -1 & 1 & -4 \\ 3 & 0 & 9 \\ 5 & -3 & 18 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$a = 3, \quad b = -1$

check: $3(1,-1,3,5) - (2,1,0,-3)$

(This is overkill but lets us practice a matrix approach.)